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A strategy for a realization of a problem free Einstein - Hilbert action along with a problem free toy cosmology

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Abstract

We describe features of a low energy effective Scalar - Tensor theory of gravity having a divergent non - minimal coupling at a point where a fermionic part of the action identically vanishes and is therefore non - dynamical. Under fairly general conditions, theories having a divergent non - minimal coupling at a point can support non - topological soliton solutions. The interior of all sufficiently large solitonic domains approach a unique limit point determined by the vanishing of the effective potential of the non - minimally coupled scalar field. This can lead to a realization of an effective Einstein - Hilbert action with no cosmological constant problem. A cosmology with characteristic features follows.

Introduction

We shall explore the possibility of realizing a problem free effective Einstein Hilbert action in a class of non - minimally coupled [NMC] theories. The cosmology that follows from such a realization is referred as a toy model on account of the limited study that we have subjected the model to.

The prospect of a dynamic geometric action forms the basis of Einstein's theory of gravitation. It is now widely accepted that the theory described by the Einstein - Hilbert [EH] action:

$$S = \int d^4x\sqrt{-g}[\frac{1}{8\pi G}R + \Lambda]$$

fares wonderfully well to classical precision tests over scales as large as hundreds of astronomical units. Here R is the Ricci scalar expressed in terms of the metric g and Λ is a (small) cosmological constant. The success of the theory is impressive enough to rule out most alternative theories at the classical level. The status of precision tests have been reviewed extensively by Will and co - workers [1992].

However, the EH action has severe theoretical inconsistencies. The appearance of a dimensional gravitational coupling constant G, and the smallness of the cosmological constant Λ, are two pathologies that impede any attempt to treat this action as a fundamental quantum theory.

The dimensionality of the gravitational coupling $[G] = M^{-2}$ $(c = \hbar = 1)$, is primarily responsible for the non - renormalizability of the theory. This is in fact a general result. Given any theory with a coupling constant with dimension M^d, a physical process is specified in perturbation theory by the amplitude of a Feynman diagram with a fixed number n of external legs. The order N [number of loops] integral at large momentum goes as $\int p^{n-Nd}dp$. For $d < 0$ (eg. -2 for gravitation), the integral diverges for any process at a sufficiently high order.

There are excellent reviews describing the cosmological constant problem (see eg. Weinberg [1989]). It is perhaps one of the most serious problems in physics. It is strictly connected with the theory of particle physics and in, some way, to Quantum Gravity. Defining $\rho_\Lambda \equiv \Lambda c^2/8\pi G$, the current critical density as $\rho_{oc} \equiv 3H^2/8\pi G$, $\Omega_\Lambda \equiv \rho_\Lambda/\rho_{0c}$ and $\Omega_o \equiv \rho_o/\rho_{oc}$ (ρ_o and H being the current matter density and Hubble parameter respectively), constraints on Λ follow from the the expression for the deceleration parameter: $q_0 = \frac{\Omega_0}{2} - \Omega_\Lambda$. With $|q_0| \sim O(1)$, one concludes

$$|\rho_\Lambda| < 2\rho_{0c} \simeq 4 \times 10^{-29} gcm^{-3} \simeq 10^{-46}\frac{m_n^4}{(\hbar/c)^3} \simeq 10^{-48} GeV^4$$

(here we have chosen the natural units: $\hbar = c = 1$ and m_n is the nucleon mass). This yields $|\Lambda| < 10^{-35} cm^{-2}$. This is an amazingly and, apparently "unnaturally", small value. It has not been possible to realize such a small value in any reasonable theoretical model. The cosmological constant was initially introduced by Einstein in an ad - hoc manner to get a static solution to the Freidmann equations. However, as soon as the expansion of the universe was established, Einstein immediately proposed the abandoning of the cosmological constant. This however turns out not to be easy. Anything that contributes to the energy density of vacuum contributes to an effective cosmological constant. Summing the zero point energies of all normal modes of any field of mass m, up to a wave number cutoff $\lambda \gg m$, yields a vacuum energy density of the order of $(\lambda)^4/(16\pi^2)$. If one believes in the validity of general relativity all the way up to the planck scale, then one would expect the cutoff to be related to the planck scale $\lambda \approx (8\pi G)^{-0.5}$. This gives $|\rho_\Lambda| \sim 2\times 10^{71}(Gev)^4$. This is off by some 120 orders of magnitude from the cosmological constraint derived earlier.

The problem is particularly acute if, as predicted by gauge theories of the fundamental interactions, the universe has undergone several phase transitions during its early history. Each phase transition produces a large change in the effective value of the cosmological constant. Be it so, the last transition is required to exit to a state of a vanishingly small cosmological constant to very high accuracy. This can normally only be achieved by an incredible fine-tuning of model parameters. The close connection of the cosmological constant problem to the inflationary model of the universe is manifestly clear. Inflation does not solve this problem; indeed, one could say that inflation is founded upon it. A positive cosmological constant in the early universe is responsible for inflation. Inflation is therefore an inevitable consequence of a theory with spontaneous symmetry breaking. However, inflation phase ought to end. The "graceful exit" is more difficult to achieve than the onset of inflation. Any description of ending inflation that does not address itself to a reasonable solution to the smallness of the residual vacuum energy (and hence a small Λ), would be seriously incomplete.

There have been several (unsuccessful) attempts to provide for a solution to the cosmological constant puzzle. We mention two approaches: (i) postulate that either a symmetry principle or quantum gravity forces the cosmological constant to vanish and (ii) postulate that the cosmological constant decays by a dynamical process.

A symmetry principle explanation would have to allow Λ to have been nonzero in the past and only become equal to zero after the last phase transition. To date there has been no satisfactory account in this direction. After the development of global supersymmetric [SUSY] theories it was realized that unbroken supersymmetry implies a vanishing vacuum energy. Quantum effects do not change this conclusion. Fermion-boson (super) symmetry leads to a neat cancellation of fermion and boson loops. In reality, however, supersymmetry has to be broken in order to account for the hierarchy of masses of particles and their supersymmetric partners. One can not construct a mechanism of SUSY breaking that could split the observed mass spectrum of supersymmetric partners while retaining the cancellation their vacuum energies. A finite vacuum energy generally survives and is of the order of m^4, m being the characteristic SUSY breaking mass scale. Thus as pointed out by Weinberg [1989], "SUSY raises the status of the problem of cosmological constant from that of a crisis to a disaster".

Attempts made by Dolgov[1982], Ford[1987] etc. fall under the second category and are based on some dynamic adjustment mechanism. For example, one could consider a new massless (or extremely light) classical field without a necessarily positive definite energy density. The interaction of this field with the curvature of space time is chosen in such a way that its energy momentum tensor would cancel out the underlying vacuum energy.

Attempts to realize the the adjustment with a scalar field proved to be unsuccessful. A free massless scalar field turns out to be stable i.e. the field equation does not possess solutions rising with time. Its stress tensor asymptotically vanishes. However, the field equation for a scalar field having a non - minimal coupling [NMC] to the scalar curvature, such as $\zeta\phi^2 R$, has unstable solutions in a de-Sitter space time. The back reaction on the de Sitter metric turns the exponential de Sitter expansion $a(t) \approx exp(\sqrt{\Lambda/3}t)$ into that of a power law expansion,$a(t) \approx t^{\sigma}$. The energy of the scalar field does cancel the vacuum energy but, with the scalar field itself not stabilizing, the effective gravitational constant $[\zeta\phi^2]^{-1}$ vanishes. Such a resolution of the cosmological problem is accompanied by the identical vanishing of the gravitational constant itself.

There have also been attempts to invoke an anthropic argument to "explain" the cosmological constant. Indeed, in studies of quantum creation of the universe, if we apply a conditional probability on all allowable cosmologies by requiring them to provide for an intelligent race to evolve, one can account for the smallness of the cosmological constant. However, one still has no means of picking out a particular small value.

Recent trends and attitudes

Over the last two decades there is an emerging consensus that the Einstein - Hilbert action ought to be regarded - at most - as a correct low energy limit of a more fundamental theory. It is believed so far that the only consistent theory of quantum gravity is the super - string theory. General considerations regarding the structure of string theory suggest that general relativity may acquire modifications, even at energies considerably lower than the Planck scale. Dilaton gravity theories in ten dimensions have recently played a prominent role in a search for a consistent effective low energy limit emerging from string inspired quantum theory of gravity [see eg. Polchinsky 1998]. This has also led to a resurrection of interest in scalar - tensor and Brans - Dicke models. A search for a suitable compactification scheme that would yield the EH action in four dimensions is on. Such a scheme has to have (amongst other requirements) provision for an appropriate fixing of the non - minimal coupling [NMC] to give the canonical gravitational constant.

A typical form for a dilaton action is described as

$$I = \int_M d^{10}x\sqrt{(-g)}[D(\phi)R + H(\phi)(\nabla\phi)^2 + V(\phi) + A(\phi)L_m$$

Here M is a ten dimensional manifold and D, H, V & A are arbitrary real functions of the dilaton field ϕ. Strictly speaking, a string inspired dilatonic gravity theory has no free dimensional constants such as may be required to specify any of these functions - in particular $V(\phi)$, $D(\phi)$. Coupling constants in string theory depend on the state and are in principle determined by the dynamics. Weyl invariance prohibits any additive dimensional constant in $V(\phi)$. A theory with a vanishing $V(\phi)$ would however lead to equivalence principle violation due to dilaton exchange. It is therefore desired that an effective potential, that would at least give the dilaton a mass and hence a finite range, ought to arise somehow. This could be expected to arise from higher order effects in string theory or even arise out of a suitably designed compactification scheme.

Once we have a prescription to get a scalar tensor [Brans - Dicke] theory with an effective potential, one has to specify an ansatz that would fix the NMC. Consider for example, a theory described by:

$$I = \int d^4x\sqrt{(-g)}[\epsilon\phi^2 R + (\nabla\phi)^2 - V(\phi) + L_m]$$

It can be seen that if $V(\phi)$ has a minimum at $\phi = \phi_o$, the theory with ϕ locked near ϕ_o is indistinguishable from an E - H theory with an effective gravitational constant $8\pi G_{eff} = (\epsilon\phi^2)^{-1}$ [Zee 1981]. There are problems with such a strategy. Firstly, it falls short of addressing itself to the cosmological constant problem as there is nothing to ensure tininess of the minimum value of the effective potential. Secondly we have not given any prescription for defining energy for a general NMC theory.

The issue of an appropriate energy definition for a general NMC has been addressed by Bose and Lohiya [1998]. Rather than describe it in isolation as a problem in its own right, in the following we shall specialize our considerations for the particular action wherein we believe a solution to the cosmological constant problem lies.

The emergent consensus of the EH action is that it ought to regarded as a truly classical theory of a classical metric field having its equations of motion determined by the standard classical variation in the presence of matter (source) fields. The EH action is not to be used to interpret the ensuing gravitational interaction in terms of propagators of gravitons or else one would land up with inconsistencies. While an ansatz that would **uniquely** provide for a realization of such a classical theory from an underlying superstring theory is yet to be discovered, we propose an alternative Brans - Dicke (scalar - tensor) theory as a candidate for an alternative classical theory. As a matter of fact, for a reader not concerned with any motivation, the following can be the starting point of my presentation:

The model:

Consider a scalar field ϕ non - minimally coupled to the scalar curvature R, through an arbitrary function $U(\phi)$, in a theory described by the action:

$$S = \int \sqrt{-g}d^4x[U(\phi)R + \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi) + L_m] \qquad (1)$$

Here $V(\phi)$ is the scalar effective potential. We shall merely require it to have a zero (not necessarily at its minima). As discussed below, throughout our analysis it would be sufficient and also appropriate for our purpose, to treat ϕ as a classical function. $V(\phi)$ is inclusive of an arbitrary constant in the theory. $L_m \equiv L_w + L_\psi$ is the contribution from the rest of the matter fields. It includes a contribution from a Dirac fermion field:

$$L_\psi \equiv \bar{U}(\phi)^{-1}[\frac{1}{2}[\bar{\psi}\overleftarrow{D}_\mu\gamma^\mu\psi - \bar{\psi}\gamma^\mu\overrightarrow{D}_\mu\psi] - m(\phi)\bar{\psi}\psi] \qquad (2)$$

Here D_μ is the spin covariant derivative (see eg. Pagels [1965])

$$\overrightarrow{D}_\mu \psi = (\partial_\mu + \Gamma_\mu)\psi$$

$$\bar{\psi}\overleftarrow{D}_\mu = (\partial_\mu \bar{\psi} - \bar{\psi}\Gamma_\mu)$$

Γ_μ are the spin connection [Fock - Ivanenko] coefficients defined by:

$$D_\nu \gamma_\mu \equiv \partial_\nu \gamma_\mu - \Gamma^\alpha_{\mu\nu}\gamma_\alpha + [\Gamma_\nu, \gamma_\mu] = 0 \tag{3}$$

The NMC $U(\phi)$ and the scaling function $\bar{U}(\phi)$ are chosen to be divergent at say $\phi = 0$. Before we proceed to describe non - trivial solutions in this model, a few words on its motivation and "naturalness" would well be in order.

We have taken the NMC $U(\phi)$ to diverge at a point that we take as $\phi = 0$. This motivates from the dynamical divergence of NMC's in classes of Brans - Dicke and scalar - tensor models, as stated before, that arise in bids to solve the cosmological constant problem. The essential distinguishing features of the theory described by eqns(1 & 2), as compared to any other NMC theory, are the presence of ϕ dependent terms multiplying the Ricci scalar and the kinetic part of a fermion field. Such terms occur as a routine in low energy limits of superstring theories (see eg. deWitt[1984]) For example, the action of $N = 1$ supergravity theory in 10 dimensions, which occurs as the so called d = 10 field theory of massless string states, has NMC term $\sim exp(\phi) \times R$ and terms ϕ raised to a negative power multiplying a spinor action.

If such features survive at sufficiently low energies where the action is suitable for classical "on - shell physics", then upon appropriate reparametrization of the scalar field ($\phi \longrightarrow \phi^{-1}$ in the above example), one can ensure that NMC diverges at (say) $\phi = 0$. The fermion action identically vanishes at this point. Fermion dynamics is thus constrained to $\phi \neq 0$ domains.

In this article we assume the possible realization of a four dimensional theory described by eqns(1 & 2) - which may arise as a classical theory out of an appropriate compactification and reparametrization of a low energy limit of a higher dimensional, decent, quantum theory. It would of course not make sense to even try and interpret such a ϕ as a quantum field, to interpret the classical action in terms of ϕ propagators, etc. Reparametrization is fine only as long as one restricts to "on - shell" classical physics and this is all that we shall need consider for our purposes.

With $U(\phi) \longrightarrow \infty$ at $\phi = 0$, flat space is a solution for $\phi = 0$ for an arbitrary $V(\phi = 0)$. This gets rid of the cosmological constant problem. However, this is a trivial cure as the blowing up of $U(\phi)$ implies a vanishing of the effective gravitational constant, and we all know that, but for gravitation, the additive constant in the action has no dynamical role in physics. What we want is a cure to the cosmological constant problem in the presence of gravitation. The least we want is non trivial solutions having ϕ non vanishing inside compact domains. The above cure for the cosmological constant would hold outside such domains while the energetics (stability requirements) of large domains would be seen below to lead to a unique limit point for ϕ in the interior.

The chosen Fermion action [eqn(2)] has an overall scaling function $\bar{U}(\phi)^{-1}$. It is sufficient to assume $\bar{U}$ to diverge at any point where U diverges. An arbitrary scaling function

$\bar{U}(\phi)^{-1}$ can be absorbed in a redefinition of the Dirac field: $\psi \longrightarrow \psi' \equiv \psi/\bar{U}^{\frac{1}{2}}$ provided $\bar{U}$ were bounded. Indeed, by such a scaling, L_ψ can be reduced to a ϕ independent Dirac lagrangian. The anti - symmetrized derivative appearing in the lagrangian, eqn(2), ensures the cancellation of the derivative of a real, non - vanishing, bounded scaling function. This result is not specific to the chosen form of the Dirac Lagrangian. Two component covariant spinor fields (for example), of arbitrary weights, can be defined as geometrical objects over a two dimensional complex space. An arbitrary, non vanishing, real scaling of such a spinor can be absorbed into a real part of the trace of a spinor connection which does not contribute to the covariant derivative of the spinor and has no interaction with other fields [Peres 1962]. At the chosen point ($\phi = 0$) where $\bar{U}(\phi)^{-1}$ vanishes, the Dirac action identically vanishes. There is no dynamics for the Dirac field at this point. The invariance of the Dirac Lagrangian under arbitrary phase change: $\delta\psi = i\epsilon\psi$ implies the vanishing divergence of the current:

$$J^\mu = \bar{U}(\phi)^{-1}\bar{\psi}\gamma^\mu\psi$$

and the conservation of the charge:

$$Q \equiv \int_\Sigma \bar{U}(\phi)^{-1}\bar{\psi}\gamma^o\psi$$

The Dirac particles would be confined to regions where $\bar{U}$ is bounded i.e. $\bar{U}(\phi)^{-1}$ is non - vanishing. The only effect of a scaling function $\bar{U}(\phi)^{-1}$ that vanishes outside a compact domain is to confine the fermions within the domain. Inside the domain the scaling can be identically absorbed in a rescaling of the fermion field. In what follows we shall essentially do this. This leads to conditions similar to those that arise in non - topological soliton solutions in the Lee - Wick model [Lee 1987]. The only difference is that in the Lee Wick model, confinement is caused by the restriction of fermions that are not on - shell outside a domain on account of the coupling with the Higgs field.

Our foremost task would be to demonstrate the vanishing of the covariant divergence of the stress tensor of the rest of the matter fields described by the action L_w. Requiring the action to be stationary under variations of the metric tensor and the fields ϕ & ψ gives the equations of motion:

$$U(\phi)[R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R] = -\frac{1}{2}[T_w^{\mu\nu} + T_\phi^{\mu\nu} + \Theta^{\mu_\nu} + 2U(\phi)^{;\mu;\nu} - 2g^{\mu\nu}U(\phi)]^{;\lambda}_{;\lambda}] \tag{4}$$

$$g^{\mu\nu}\phi_{;\mu;\nu} + \frac{\partial V}{\partial\phi} - R\frac{\partial U}{\partial\phi} + \frac{\partial m}{\partial\phi}\bar{\psi}'\psi' = 0 \tag{5}$$

$$\gamma^\mu D_\mu\psi' + m(\phi)\psi' = 0 \tag{6}$$

$$D_\mu\bar{\psi}'\gamma^\mu - m(\phi)\bar{\psi}' = 0 \tag{7}$$

Here $T_w^{\mu\nu}$, Θ^μ_ν are the energy momentum tensors constructed from L_w and L_ψ respectively, and

$$T_\phi^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}[\frac{1}{2}\partial^\lambda\phi\partial_\lambda\phi - V(\phi)] \tag{8}$$

L_w is taken to be independent of ϕ. The two Dirac equations imply L_ψ to be null. The scalar field equations are therefore independent of $\bar{U}(\phi)$. The Fermion stress tensor is simply:

$$\Theta^\mu_\nu \equiv -\frac{1}{2}[\bar{\psi}'\overleftarrow{D}_\nu \gamma^\mu \psi' - \bar{\psi}'\gamma^\mu \overrightarrow{D}_\nu \psi'] \tag{9}$$

When applied to any spinor or any "spin - matrix" such as the Dirac matrices, one replaces the ordinary derivative by the spin - covariant derivative (Pagels [1965]). The covariant divergence of (9) is easily seen to reduce to:

$$\Theta^\mu_{\nu;\mu} = \frac{\partial m}{\partial \phi}\partial_\nu \phi \bar{\psi}'\psi' \tag{10}$$

Thus there is a violation of equivalence principal as far as the fermi field is concerned. However, in a region where the scalar field gradient vanishes or for a ϕ independent fermion mass, the covariant divergence of the fermion field stress tensor vanishes. To see how the equivalence principal strictly holds for the rest of the matter fields, i.e.: $T^{\mu\nu}_{w;\nu} = 0$, consider the covariant divergence of (4). From the contracted Bianchi identity satisfied by the Einstein tensor, we obtain

$$U(\phi)_{,\nu}[R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R] = -\frac{1}{2}[T^{\mu\nu}_{w;\nu} + t^{\mu\nu}_{;\nu} + \Theta^{\mu\nu}_{;\nu}] \tag{11}$$

with

$$t^{\mu\nu} \equiv T^{\mu\nu}_\phi + 2U(\phi)^{;\mu;\nu} - 2g^{\mu\nu}U(\phi)^{;\lambda}_{;\lambda} \tag{12}$$

Using the identity: $U(\phi)^{;\rho}R_{\rho\alpha} = U(\phi)^{;\lambda}_{;\lambda;\alpha} - U(\phi)^{;\lambda}_{;\alpha;\lambda}$ and eqn(10), eqn(11) reduces to

$$-\frac{1}{2}U(\phi)^{,\mu}R = -\frac{1}{2}[T^{\mu\nu}_{w;\nu} + T^{\mu\nu}_{\phi;\nu} + \partial^\mu m\bar{\psi}'\psi']$$

Finally, using the equation of motion for the scalar field (5), all the ϕ dependent terms cancel the left hand side - giving the vanishing of the covariant divergence of the (w-) matter stress energy tensor.

Newt, we find the expression for a conserved pseudo energy momentum tensor [Bose, Lohiya 1999]. Defining

$$\mathrm{A} \equiv \sqrt{-g}g^{\sigma\rho}[\Gamma^\alpha_{\sigma\rho}\Gamma^\beta_{\alpha\beta} - \Gamma^\alpha_{\beta\rho}\Gamma^\beta_{\alpha\sigma}] \tag{13}$$

$$\mathrm{B} \equiv [U\mathrm{A} - \sqrt{-g}g^{\sigma\rho}\Gamma^\alpha_{\sigma\alpha}U_{,\rho} + \sqrt{-g}g^{\sigma\rho}\Gamma^\alpha_{\sigma\rho}U_{,\alpha}] \tag{14}$$

and $\hat{\mathrm{B}} \equiv \mathrm{B} + \sqrt{-g}L_{\phi+\psi}$, gives the expression for the pseudo energy momentum vector:

$$P_\mu \equiv \int_\Sigma d\Sigma[\sqrt{-g}T^o_{w\mu} - \hat{\mathrm{B}}\delta^o_\mu - \frac{\partial \hat{\mathrm{B}}}{\partial g^{\tau\beta}_{,o}}g^{\tau\beta}_{,\mu} - \frac{\partial \hat{\mathrm{B}}}{\partial \phi_{,o}}\phi_{,\mu} - \bar{\psi}_{,\mu}\frac{\partial \hat{\mathrm{B}}}{\partial \bar{\psi}_{,o}} - \frac{\partial \hat{\mathrm{B}}}{\partial \psi_{,o}}\psi_{,\mu}] \tag{15}$$

which is conserved on a spacelike hypersurface Σ.

As noted earlier, with choice of NMC function $U(\phi)$ diverging at $\phi = 0$, a globally flat space is a solution for an arbitrary $V(\phi)$ at a vanishing ϕ. For the above expressions of the energy - momentum pseudo vector to make sense, one has to subtract the contribution from a "reference action". The contribution from such a reference action is determined by the asymptotic behaviour of the fields on a classical solution [Hawking and Horowitz [1996], Bose and Lohiya [1999]). We shall be interested in all solutions that would be identically flat outside compact regions of a 3 - dimensional space. We shall therefore require a globally flat spacetime to have a vanishing energy and to achieve this it is sufficient to choose the reference action to be:

$$S_o \equiv -\int \sqrt{-g} d^4x [V(0)] \tag{16}$$

This merely amounts to adding a constant to the scalar potential in eqn(1) that ensures its vanishing for $\phi = 0$. In what follows we assume this to have been done. In other words we shall consider the theory defined by eqns(1) and (2) with $V(0) = 0$. This has nothing to do with the cosmological constant problem. Globally flat space is a solution to the theory on account of the diverging NMC at $\phi = 0$. By choosing a vanishing effective potential $V(0)$, one has merely ensured the finiteness of the energy of this globally flat space solution.

We shall be interested in a non - trivial configuration that has ϕ vanishing outside, and $N_f >> 0$ ψ fermions trapped inside, a compact domain. With the fermion number conserving, the minimum energy configuration of such solutions would be non - topological soliton solutions [NTS's] in the theory. Properties of such an NTS are similar to the Lee - Wick [1987] constructs. Lee and Wick considered a theory with a Lagrangian:

$$L = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + \bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}[1 - \frac{\phi}{\phi_o}]\psi \tag{17}$$

where

$$V(\phi) = \frac{1}{2}m_\phi^2\phi^2[1 - \frac{\phi}{\phi_o}]^2 \tag{18}$$

With Fermion number conserved in this theory, solutions representing a degenerate distribution of fermions with a total fermion number N_f, at $\phi = \phi_o$ would remain confined there if the energy of a fermion in less than its on - shell energy at $\phi = 0$. The NTS is a ball of radius r_b inside which the scalar field is locked at $\phi = \phi_o$. Across the surface of the ball of thickness $\sim m_\phi^{-1}$, the scalar field transits to $\phi = 0$. As a matter of fact the spherical shape of the distribution follows from degeneracy of the fermion ensemble. For a degenerate Fermion distribution, the fermion density is determined in terms of the chemical potential that occurs as a lagrange multiplier for the conserved fermi number of the ensemble. Therefore the volume is determined for a fixed conserved number of degenerate fermions. A positive definite surface energy would break the degeneracy of volume preserving deformations of the distribution - choosing a configuration of minimum surface area. This gives the minimum energy configuration to be a sphere. In terms of the chemical potential μ, the fermion density goes as $\rho_f \sim \mu^4$. The fermion number density goes as $n_f \sim \mu^3$. With the total energy and fermion number: $E_f = \rho_f v \sim \mu^4 r_b^3$, $N_f = n_f v \sim \mu^3 r_b^3$, the chemical potential eliminates to give $E_f \sim N_f^{4/3}/r_b$. For a large enough r_b, the surface tension is given by the expression: $s \sim m_\phi \phi_o^2/6$, the total surface energy being $E_s = 4\pi s r_b^2$. For a given N_f, one may vary the radius to seek the minimum of the volume and surface energies to give $E_f = 2E_s$. The mass of the soliton is expressed as $M = E_f + E_s = 3E_s = 3E_f/2 = 12\pi s r_b^2$. This allows one to express:

$$N_f \sim s^{3/4} r_b^{9/4}; \quad M \sim s^{1/3} N_f^{8/9}; \quad r_b \sim s^{-1/3} N_f^{4/9} \tag{19}$$

Thus with a given surface tension, one could get arbitrarily massive NTS's by increasing the fermion number. However, the above flat spacetime analysis can not be extended past the Schwarzschild limit. In the Lee - Wick model, the Schwarzschild limit was determined as the criteria for stability against gravitational collapse. In our case we shall use this limit as a benchmark to keep sufficiently clear - off as a convenience in order that the flat spacetime analysis holds. This limit is simply obtained by equating the radius of the NTS to the Schwarzschild radius for a critical mass ball M_c: equating $r_b = 2GM_c$, and with the expression of mass of a ball, one gets $M_c \sim (48\pi G^2 s)^{-1}$. For $s \sim (30Gev)^3$, M_c is roughly $10^{15} M_\odot$ with the critical radius some 10^2 Pc.

In what follows, we shall be interested in NTS's as large as some 100 to 1000 KPc. We shall also like to explore a dynamic generation of of a class of low mass fermions in a Hot big bang cosmology. Any species of particles that decoupled from equilibrium when it was relativistic, ought to have the same relic density as say the relic photons or neutrinos: some $\sim$ 200 per cc. To get $\sim 10^3$ KPc NTS, it would suffice to have $s \sim (Mev)^3$. This gives $N_f \sim 10^{72}$ to 10^{75} with the mass of the ball some 12 orders of magnitude smaller than the the Schwarzschild (critical) mass for $s \sim (Mev)^3$. We conclude that for such a surface tension and for NTS's even as large as a Mega parsec, one can consistently ignore the spacetime curvature in such solutions. We shall do this whenever convenience demands.

As discussed before, ψ' can be replaced by ψ in any region where the scaling function is bounded. Making this replacement, it is easy to follow (Sethi [1999]) to demonstrate the existence of non - trivial solutions. In the "weak (gravitational) field approximation", that would justify retaining only a first order deviation from a flat metric, the metric can be expressed in terms of the spherical [Schwarzschild] coordinates:

$$ds^2 = e^{2u} dt^2 - e^{2\bar{v}} dr^2 - r^2 [d\theta^2 + sin^2\theta d\varphi^2] \tag{20}$$

We look for a solution describing the scalar field trapped to a value $\phi = \phi_{in}$ in the interior of a sphere of radius r_b and making a transition across a thin surface to $\phi = 0$ outside. The fermi gas trapped inside the soliton is described, in the Thomas Fermi approximation (Lee, Pang [1987]), by the familiar distribution in momentum space: $n_k = \theta(k - k_f)$, k being the momentum measured in an appropriate local frame that depends on r and k_f the fermi momentum. The fermion energy density is given by:

$$W = \frac{2}{8\pi^3} \int d^3k n_k \epsilon_k \tag{21}$$

with $\epsilon_k = \sqrt{k^2 + m(\phi_{in})^2}$. The fermion number density ν_f and the non - vanishing components of fermion stress energy tensor are:

$$\nu_f = \frac{2}{8\pi^3} \int d^3k n_k \tag{22}$$

$$T^t_t = W$$

$$T^r_r = T^\theta_\theta = T^\varphi_\varphi = T^\rho_\rho \equiv -T = -\frac{2}{8\pi^3}\int d^3k n_k \frac{k^2}{3\epsilon_k} \tag{23}$$

The trace of the stress tensor is just:

$$T^\mu_\mu = W - 3T = m(\phi_{in})S \tag{24}$$

with S the scalar density:

$$S = \frac{2}{8\pi^3}\int d^3k \frac{n_k}{\epsilon_k} m(\phi_{in}) \tag{25}$$

Defining $G_{in} \equiv U(\phi_{in})^{-1}$ as the effective interior "gravitational constant", the metric field equation in the interior can be expressed in the above spherical coordinates as:

$$r^2 G^t_t = e^{-2\bar{v}} - 1 - 2e^{-2\bar{v}} r \frac{d\bar{v}}{dr} = -8\pi G_{in} r^2 [W + V(in)] \tag{26}$$

$$r^2 G^r_r = e^{-2\bar{v}} - 1 + 2e^{-2\bar{v}} r \frac{du}{dr} = 8\pi G_{in} r^2 [T - V(in)] \tag{27}$$

$$r^2 G^\theta_\theta = e^{-2\bar{v}} [r^2 \frac{d^2u}{dr^2} + [1 + r\frac{du}{dr}] r \frac{d}{dr}(u - \bar{v})] = 8\pi G_{in} r^2 [T - V(in)] \tag{28}$$

The scalar field satisfies:

$$\phi^{;\mu}_{;\mu} + V' + m'(\phi)S - U'R = 0 \tag{29}$$

Taking the trace of the Einstein tensor G^μ_μ, the Ricci scalar R substitutes in eqn(29) to give the following radial equation for the scalar field in the weak field limit:

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} + F(U)[\frac{d\phi}{dr}]^2 = \frac{dW}{d\phi} \tag{30}$$

where

$$\frac{dW}{d\phi} \equiv [V' - \frac{U'}{U}(mS + 4V)]/[1 - \frac{3U'^2}{U}]$$

and

$$F(U) \equiv \frac{U'}{U}(\frac{1}{2} - 3U'')/[1 - \frac{3U'^2}{U}]$$

Sufficient conditions for existence for an NTS are: $U, U', U'' \longrightarrow \infty$, $|V| < \infty$ as $\phi \longrightarrow 0$; there is an open domain containing the zero of $V(\phi)$ in which $U, U', U'', U'/U$ are sufficiently large, and finally

$$\int_\epsilon^\phi F[U(\phi)]d\phi \longrightarrow \infty \quad iff \quad \epsilon \longrightarrow 0 \tag{31}$$

The profile of U, V and W are displayed in Figure 1. The condition on F, eqn(31), ensures that for a large enough radius, a “bounce” solution to eqn(30), in an equivalent one dimensional particle problem in the inverted potential $-W$, makes its way all the way upto $\phi = 0$.

The form for the metric in the linear approximation follows from eqns(26 - 28). Defining $\hat{C} \equiv 8\pi G[W + V(\phi_{in})]$, we get $\bar{v} = -\hat{C}r^2/6$. The expression for u follows from:

$$2e^{-\bar{v}}r[\frac{du}{dr} + \frac{d\bar{v}}{dr}] = 8\pi G r^2[T + W] \equiv \tilde{C}r^2 \tag{32}$$

Whence,

$$\frac{du}{dr} + \frac{d\bar{v}}{dr} = \frac{1}{2}\tilde{C}r \tag{33}$$

From the expression derived for $\bar{v}$, we get:

$$u = u_o + \frac{r^2}{2}[\frac{\tilde{C}}{2} + \frac{\hat{C}}{3}] \tag{34}$$

u_o is a small constant that determines the rate at which clocks tick inside the soliton in comparison to the exterior.

With the spacetime almost flat for an NTS well away from the Schwarzschild bound, the solution has the scalar field locked to a constant in the interior of a spherical domain and transits across $r = r_b$ to zero across the boundary with thickness much less that r_b. The pressure of the degenerate fermions keeps the soliton from squashing. The total energy at a general point ϕ in the interior is expressible as the sum of (i) the volume energy, (ii) the surface energy and (iii) the energy of the fermions:

$$E = \frac{4\pi}{3}V(\phi_{in})r_b^3 + 4\pi s r_b^2 + \alpha\frac{N_f^{4/3}}{r_b}; \quad \alpha \equiv \frac{1}{2}(\frac{3}{2})^{5/3}\pi^{1/3} \sim O(1) \tag{35}$$

The profile of the three terms are exhibited in figure 2. The alternate profiles of the first term (i); (i’) depend on the sign of V being positive or negative respectively. For a vanishing V, the minimum of (ii) and (iii) give the total energy $E \sim N_f^{8/9}(4\pi s)^{1/3}$, with an energy per fermion $\sim (4\pi s)^{1/3}N_f^{1/9}$. For $V(\phi_{in}) > 0$, large r_b, N_f solutions have energy $E \sim 4(4\pi V)^{1/4}N_f/3$, with the energy per fermion roughly independent of the number of fermions. Finally, for negative $V(\phi)$, NTS’s exist only for a small fermion number and for small enough $|V|$. The existence require the surface term to dominate for small r_b, $\bar{N}_f$ solutions. The energy per fermion is again $\sim (4\pi s')^{1/3}/\bar{N}_f^{1/9}$. One thus concludes that for large enough fermion number, the minimum energy NTS’s are the ones that have the scalar field approaching the zero ϕ_o of the effective potential.

Thus given a distribution of NTS’s which are allowed to accrete, (with the fermion pressure being capable of holding the scalar field at an arbitrary point in an open interval containing the zero of the effective potential), all sufficiently large NTS’s would have the scalar field in their interior approaching a limit point viz.: the zero of the effective potential. Thus all large NTS's have ϕ approaching ϕ^o_{in} in their interiors. The effective gravitational constant inside all large NTS's dynamically approach $[U(\phi^o_{in})]^{-1}$. The effective cosmological constant goes to zero inside large NTS's and is identically zero outside where $U(\phi \longrightarrow 0)$ has been chosen to diverge. This sorts out the cosmological constant problem.

Unfortunately, in the above argument we have only considered NTS's for which the interior spacetime curvature effects can be neglected. We have provided a justification for NTS's near $V \sim 0$ for which some 100 to 1000 Kpc NTS with a surface tension $\sim Mev^3$ has negligible curvature effects. What one requires to establish is that a general NTS, with an arbitrary number of fermions trapped in the interior, has a larger value for the energy per fermion than the corresponding value in a large enough NTS at ϕ at the zero of the effective potential. The inherent non - linearity of the problem is the biggest impediment for such a study. The same reason makes it difficult to establish a "spherical rearrangement theorem (Coleman [1978]) for this model. These issues ought to be covered - especially because the ensuing cosmology appears to be fairly promising. In what follows we take the stability of a large NTS near the zero of the potential in general as a conjecture and describe features of an ensuing toy cosmology.

Features of a toy cosmology

In the model described above, a divergent NMC can provide for a vanishing cosmological constant. Non - trivial solutions for the scalar field can provide for gravitating pockets inside compact domains - with the divergent NMC being restricted to their exterior. A value $N_f \approx 10^{75}$ for $s \approx (Mev)^3$ would give a NTS of a size of tens of kilo parsecs. Such an N_f is of the same order as the relic background neutrinos / photons in the universe. Thus a fermion species that decouples very early in the universe, would be sufficient to provide N_f for gravitating domains as large as a Halo of a typical large structure (galaxy / local group etc.).

With the effective gravitational constant identically vanishing outside a NTS, one could conceive of a cosmological model that starts with a hot big - bang and evolves as a Milne [1935] model from its birth. The hot big bang would be required to produce the fermions confined to non - vanishing scalar field domains. These NTS's would then evolve to the current distribution of gravitating domains by mutually colliding and coalescing.

On large scales, the universe evolves with the Freidmann - Robertson - Walker scale factor increasing linearly with time: $a(t) = t$. This has characteristic features: (i) With $\int_o^t dt/a(t)$ unbounded for any $t > 0$, there is no horizon problem in the theory. (ii) With the expansion parameter not determining any "critical - density" in the model, there is no flatness (fine tuning) problem. (ii) Concordance with the standard classical cosmological tests, viz.: the number count, angular diameter and the luminosity distance variation with redshift. The first two tests are quite sensitive to models of galactic evolution and for this reason have (of late) fallen into disfavour as reliable indicators of a viable model. However the magnitude - redshift measurements on SN 1A have a great degree of concordance with $\Omega_\Lambda = \Omega_M = 0$ (Sethi [1997, 1999]). (iii) With the scale factor evolving linearly with time, the Hubble parameter is precisely the inverse of the age t. Thus the age of the universe inferred from a measurement of the Hubble parameter is 1.5 times the age inferred by the same measurement in standard matter dominated model. Such a cosmology promises consistency with an older universe. (iv) The deceleration parameter is predicted to vanish. (v) early universe nucleosynthesis is not ruled out in a Milne universe. It has been shown [Batra 1999, Sethi 1999] that for a baryon entropy ratio $\eta \approx 5 \times 10^{-9}$, one gets $\sim 24\%$ of helium-4 and metallicity quite close to that observed in type II stars and low metallicity clouds. The cosmology comes with its characteristic predictions and may well be distinguishable by the next generation of experiments.

Conclusions

What we have profiled is a plausible program with an exploratory spirit. We had been looking for a framework that could provide for a spatial variation of the effective gravitational constant as a solution to the cosmological constant problem. It was this search that attracted our attention to remarkably encouraging results in the theory of non - topological solitons [NTS's] in the Lee - Wick model. Given a (large), conserved fermion number and a given surface tension of such a soliton, the smaller the magnitude of the interior volume energy density (given by the value of the effective potential), the larger is the size of the NTS. Therefore, given a distribution of NTS's in a Lee Wick model which are allowed to accrete, (with the fermion pressure being capable of holding the scalar field at an arbitrary point in an open interval containing the zero of the effective potential), all sufficiently large NTS's would have the scalar field in their interior approaching a limit point viz.: the zero of the effective potential. To this known [26,35] result we have only added a non - minimal coupling [NMC] to translate it to read that the effective gravitational constant inside all large domains would dynamically tend to a universal value with $V_{eff} \longrightarrow 0$ in the interior. If in addition we have the NMC diverging at any point - which we can, with no loss of generality, take as $\phi = 0$ - we would get a framework in which the cosmological constant problem is resolved.

We have demonstrated that in a whole class of scalar tensor theories in which the non - minimal coupling diverges and for which the classical effective potential vanishes at some point, classical scalar field condensates can occur as NTS's. The effective gravitational constant inside all large domains would approach a universal value and the effective cosmological constant would drift to zero. The dynamical tuning of the effective cosmological constant to a small value and the effective gravitational constant to a universal value are compelling features - enough to explore the possibility of raising the toy model described here to the status of a viable cosmology.

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References

1. C. M. Will; [1992] “The confrontation between General Relativity and experiment, A 1992 update”, WUGRAV - 92
2. S. Weinberg, Rev. Mod. Phys 1 (1989)
3. A. D. Dolgov, in ”The very Early Universe” eds. G. W. Gibbons, S. W. Hawking and S. T. Siklos, (1982) CUP.
4. L. H. Ford, ”Phys Rev $\underline{D35}$ 2339, [1987]
5. J. G. Polchinski, “String Theory”, Cambridge Univ. Press, [1998]
6. A.Zee,in 1981 Erice Conference Proceedings,ed. A.Zichichi, Plenum.
7. S. Bose, D. Lohiya, “Behaviour of Quasilocal mass under Conformal transformations”Phys Rev $\underline{D59}$ 044019
8. H. Pagels, Ann. Phys. 31, 64 (1965)
9. B. deWitt, P. Fayet, M. Grisaru, “Supersymmetry, Supergravity and Superstrings”, World Scientific, p 37, [1986]
10. T. D. Lee, Phys. Rev. $\underline{D35}$, 3637 (1987); T. D. Lee & Y. Pang, phys. Rev. $\underline{D36}$, 3678 (1987);T.D. Lee and G.C.Wick, Phys.Rev. $\underline{D9}$, 2291 (1974)
11. Peres A, Nuovo, Cimento (Supp)24, 389 [1962]
12. S. W. Hawking, G. T. Horowitz, Class. Quantum Grav. 13 1487 [1996]
13. M. Sethi, D. Lohiya, “Aspects of a Coasting Universe”, Univ. Delhi preprint, 1996 [also GR15 proceedings 1997]; Class. Quantum Grav. 16 1 [1999]
14. E.A.Milne Relativity,Gravitation and World Structure (Oxford) (1935)
15. A. Batra, D. Lohiya, “Nucleosynthesis in a simmering universe” GR 15 proceedings [1997], gr-qc/9808031.
16. S. Coleman, V. Glaser, A. Martin; Comm. Math. Phys. 58, 211 [1978]